Diffraction

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Note : The notes given in this file is no substitute to the much detailed discussion held in the online/contact classes with active participation of students. It , at best, serves the purpose of ready reference for important concepts/derivations covered in the classes.

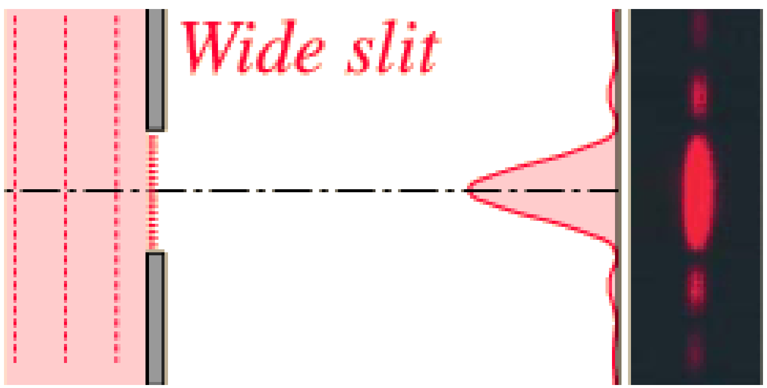
Diffraction of light

- Phenomenon
- Conditions for Max and Min
- Resolution of eye
- Resolution of microscope
- Resolution of telescope
- Validity of ray optics

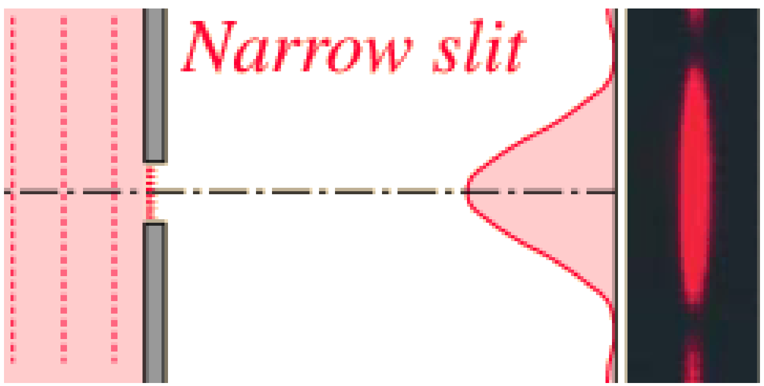
Diffraction of light

The phenomenon of bending of light around the edges of obstacles or apertures is called diffraction of light

Effects of diffraction become significant when the size of object (or aperture) is comparable to wavelength of light.



Large aperture width – Less spread of light in geometrical shadow.



Small aperture width – More spread of light in geometrical shadow.

Diffraction of light

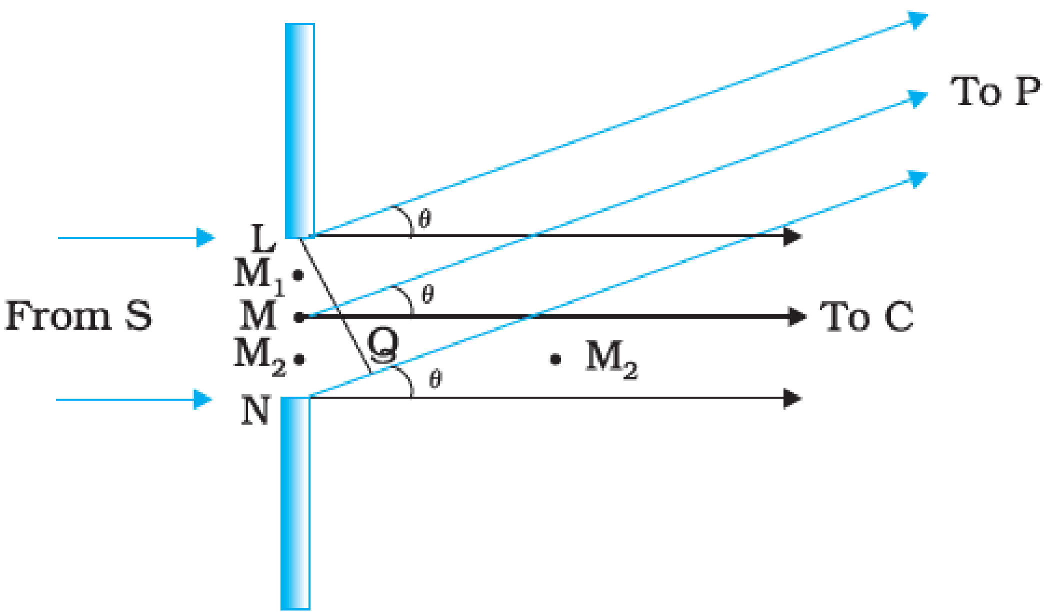
The phenomenon of diffraction can be understood by considering superposition of waves emanating from different parts of a single slit. As waves from these parts reach a point on the screen the resultant is given by the path difference between them.

Condition for maximum is

$$\theta = \frac{(2n+1)\lambda}{2a}$$

Condition for minimum is

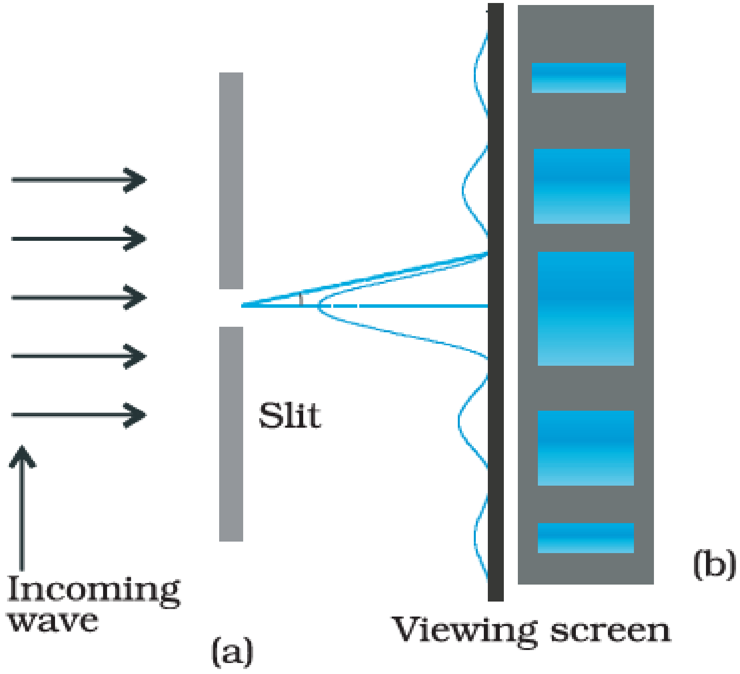
$$\theta = \frac{n\lambda}{a}$$



Fringe pattern in diffraction

Diffraction pattern has a central bright maximum which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre, on either side.

Interference pattern is calculated by superposing two waves originating from the two narrow slits. Diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.



Click here for simulation

For a single slit of width a, the first null of the interference pattern occurs at an angle of λ/a. At the same angle of λ/a, we get a maximum (and not a minimum) for two narrow slits separated by a distance a.

Additional detail *

Diffraction occurs due to superposition of waves from different parts of the unblocked wavefront. Mathematical treatment involves successive addition of amplitudes of adjacent waves to obtain net amplitude (and therefore intensity) at any point on the screen.

A detailed derivation of distribution of intensity requires the use of an integration. However using a continuous addition approach, resultant intensity on the screen at an angle θ, due to an aperture of width a, is given by

$$I = I_o \left[\frac{\sin^2 \beta}{\beta^2} \right] \qquad \text{where } \beta = \frac{\pi \, a \sin(\theta)}{\lambda}$$

From the above set of equation, with a little bit of effort one can observe that

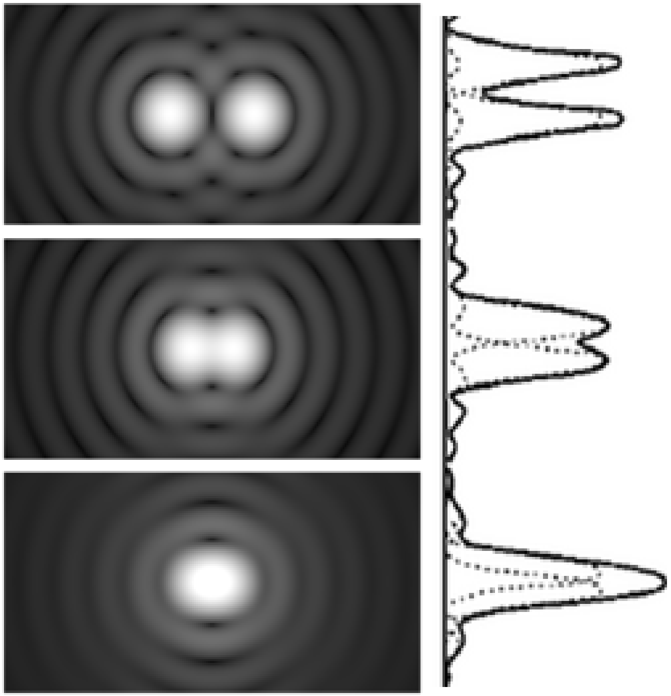
(a) Central fringe is the brightest
(b) Width of the central bright fringe
(c) Intensity decreases for fringes of higher order
(d) Fringe width is not constant

Resolution and optical instruments

In an optical instrument, light enters through the objective which acts as a aperture (single slit) resulting in effects of diffraction in the image formed.

Due to diffraction, image of a point object consists of a primary image surrounded by alternate bright and dark 'bands'.

Rayleigh's criterion : Two objects are said to be *just resolve*d if first minimum of image of one object falls on the central maximum of image of the other object.



Images overlapping due to diffraction

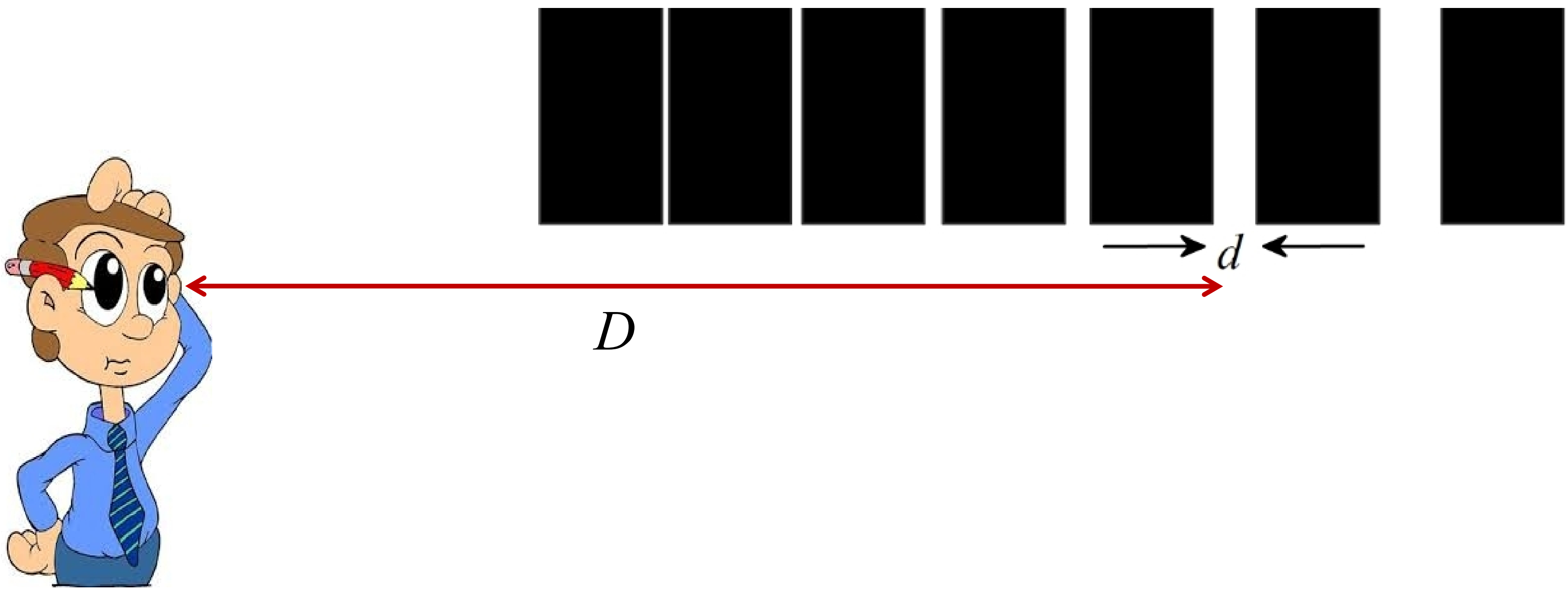
Diffraction peaks

Resolution of human eye

D is distance between person and the screen

d is the smallest observable gap between two black strips

$$resolution = \frac{d}{D}$$



Resolving power of a telescope

Objective of a telescope is similar to a circular aperture. As light waves pass through the aperture they undergo diffraction. The image therefore consists of a central bright band followed by subsequent bright and dark bands as observed in case of single slit diffraction.

(a) (b) (c)

Radius of the central bright band is given by the relation

$$r_o = 1.22\frac{f\lambda}{2a}$$

f : focal length

λ : wavelength

$2a$: diameter of objective lens

As the image is formed in the focal plane of the lens, the condition for clear resolution of the images is

$$f\Delta\theta \approx r_o \approx 1.22\frac{f\lambda}{2a}$$

$$\Rightarrow \Delta\theta \approx 0.61\frac{\lambda}{a}$$

- Smaller value of $\Delta\theta$ implies a lesser spread (higher resolution) in the image.
- Resolution can be increased by increasing the aperture of objective lens.

Resolving power of a microscope

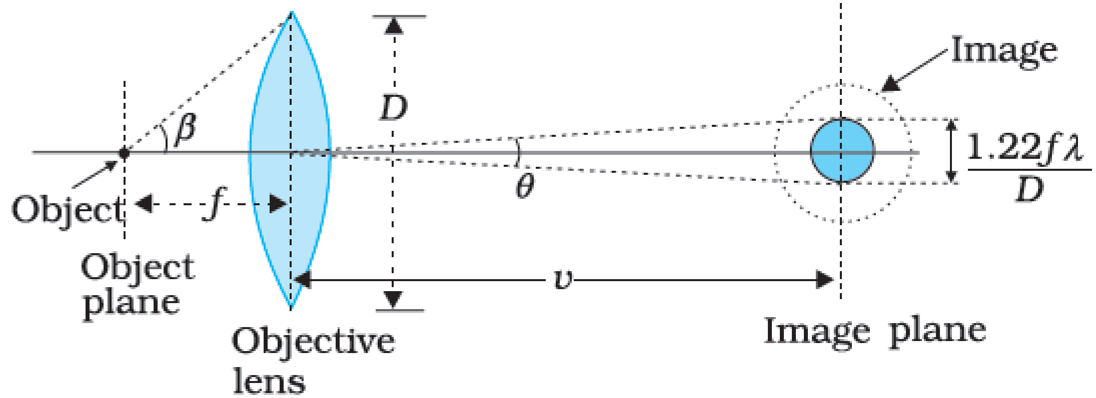
Consider an object placed just beyond the focal plane (f) of the lens. Image is formed at a distance v from the lens. Let D be the aperture of the lens and λ be the wavelength of light used.

Angular spread in the image due to diffraction is given by the relation

$$\Delta\theta = 1.22\frac{\lambda}{D}$$

Image is formed at v. Linear spread in the image due to diffraction is therefore

$$\Delta v = v \times 1.22\frac{\lambda}{D}$$



Corresponding minimum distance of separation between two object should therefore be

$$d_{\min} = \left(v \times 1.22\frac{\lambda}{D}\right)/m$$

$$d_{\min} = \left(1.22\frac{\lambda}{D}\right)\frac{v}{m}$$

$$d_{\min} = \left(1.22\frac{\lambda}{D}\right)f \qquad \left(\because m \approx \frac{v}{f}\right)$$

From figure $\tan(\beta) \approx \frac{D}{2f}$

Resolving power of a microscope

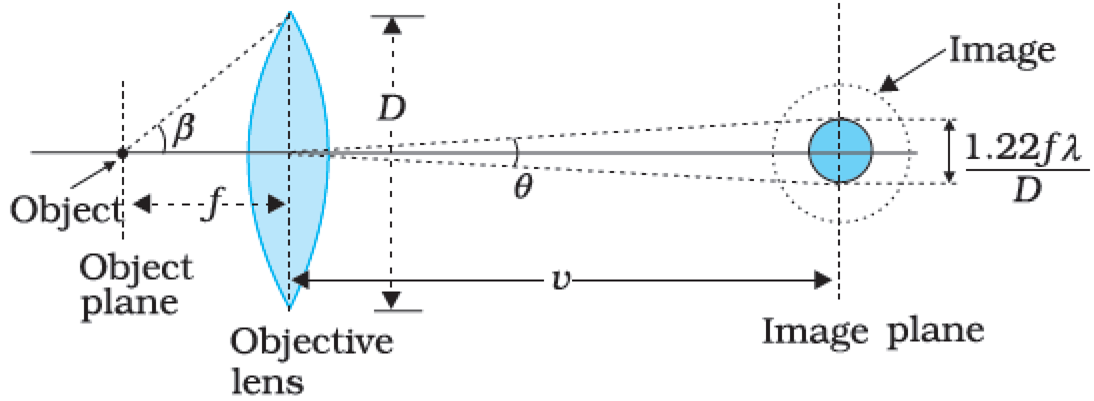
$$d_{min} = \frac{1.22\lambda}{2\tan(\beta)}$$

$$d_{min} \approx \frac{1.22\lambda}{2\sin(\beta)}$$

If the medium between the object and the objective lens is not air but a medium of refractive index n

$$d_{min} \approx \frac{1.22\lambda}{2n\sin(\beta)}$$

- $n \sin(\beta)$ is called numerical aperture.
- Smaller value of d_{min} implies better resolution.



- resolving power can be increased by choosing a medium of higher refractive index
- resolving power of a microscope is basically determined by the wavelength of the light used

Validity of ray optics

Effects of diffraction become significant when the size of object (or aperture) is comparable to wavelength of light.

Angular spread in the image due to diffraction is given by the relation

$$\theta \approx \frac{\lambda}{a}$$

In travelling a distance z, the width in the diffracted beam is given by

$$\Delta x \approx z\frac{\lambda}{a}$$

Effects of diffraction become significant if the spread in the beam is comparable to the size of the object/aperture. Therefore

$$a \approx z\frac{\lambda}{a}$$

A quantity Z_F called the *Fresnel distance* is defined by the following equation

$$z_F \approx \frac{a^2}{\lambda}$$

for distances much smaller than z_F , spreading due to diffraction is smaller compared to the size of the beam and therefore approximation of ray optics is valid.